classmate

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If a l.t. from $T: V_1 \to V_2$ it is one-one and ONTO i.e. BIJECTIVE.

It is called ISOMORPHISM ($V_1 \cong V_2$)

There always exist atleast one l.t. betn any two v.s. say V_1 & V_2 over the same field.

#Q1: Let $T: \mathbb{R}^2(\mathbb{R}) \to \mathbb{R}^2(\mathbb{R})$ be a map. Then, which of these foll. is/are L.T.

(A) $T(x,y) = (x,y)$ ✓

$T(\alpha a + b) = T(\alpha(x_1,y_1) + (x_2,y_2))$ $\left(\begin{matrix} a = (x_1, y_1) \\ b = (x_2, y_2) \end{matrix}\right)$

$= T((\alpha x_1 + x_2, \alpha y_1 + y_2))$

$= (\alpha(x_1) + x_2, \alpha y_1 + y_2) = (\alpha x_1, \alpha y_1) + (x_2, y_2)$

$= \alpha(x_1, y_1) + (x_2, y_2)$

$= \alpha T(a) + T(b)$

$\therefore$ L.t.

(B) $T(x,y) = (x,0)$ ✓

$T(\alpha a + b) = T(\alpha(x_1,y_1) + (x_2,y_2))$

$= T(\alpha x_1 + x_2, \alpha y_1 + y_2) = (\alpha x_1 + x_2, 0)$

$= (\alpha x_1, 0) + (x_2, 0) = \alpha T(a) + T(b)$

$\therefore$ l.t.

(C) $T(x,y) = (x^2, y)$ ✗

$T(\alpha a + b) = T(\alpha(x_1,y_1) + (x_2,y_2)) = T((\alpha x_1 + x_2), (\alpha y_1 + y_2))$

$= ((\alpha x_1 + x_2)^2, \alpha y_1 + y_2) = (\alpha^2 x_1^2 + x_2^2 + 2x_1x_2, \alpha y_1 + y_2)$

$\alpha T(a) + T(b) = \alpha(x_1^2, y_1) + (x_2^2, y_2) = (\alpha x_1^2 + x_2^2, \alpha y_1 + y_2)$ not L.T.